Part 10: Static Equilibrium

Physics for Engineers & Scientists (Giancoli): Chapters 9
University Physics V1 (Openstax): Chapter 12

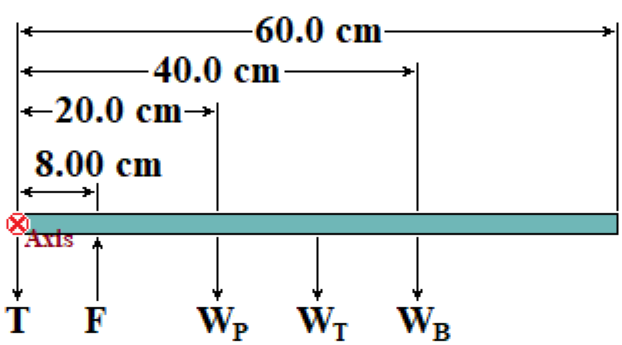
Statics

- To be in static equilibrium the net force on an object must be zero. In addition, the net torque must be zero as well.

$$\sum \vec{F} = 0 \quad \rightarrow \quad \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \qquad \sum \vec{\tau} = 0$$

- Torque and moment of inertia are both defined about some axis of rotation. What do we use as an axis of rotation?
    - A static object is not accelerating about any axis. Therefore, the net torque about every axis is zero.
    - This gives us the freedom to select a convenient axis of rotation.
        - If we choose the point where a force acts to be the axis of rotation, then that force creates no torque (the lever arm is zero). This force will not show up in a torque equation.
        - A clever choice of axis of rotation can be used to remove a variable from your equation.

Example: A student carries a 60.0 cm long lunch tray with a single hand. To do so her fingers press upwards 8.00 cm from the left edge of the tray, and her thumb presses downward on the left edge. The mass of the lunch tray is 0.100 kg. They tray holds a bowl of soup of mass 0.500 kg whose center of mass sits 40.0 cm from the left edge and a plate of food of mass 0.750 kg whose center of mass sits 20.0 cm from the left edge. Determine the forces exerted by the student's thumb and fingers.



$$W_T = m_T g = (0.100\ kg)\left(9.80\frac{m}{s^2}\right) = 0.980\ N$$

$$W_B = m_B g = (0.500\ kg)\left(9.80\frac{m}{s^2}\right) = 4.90\ N$$

$$W_P = m_P g = (0.750\ kg)\left(9.80\frac{m}{s^2}\right) = 7.35\ N$$

Setting the sum of forces equal to zero will give us 1 equation with 2 unknowns. Consequently we will need to use a torque equation.

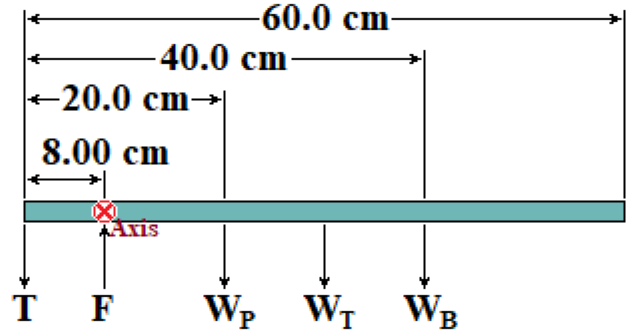
To start we will use the left edge as the axis of rotation. The force generated by the thumb (T) generates no torque about this axis and will disappear from our equations leaving just F.

$$\sum \tau = (8.00\ cm)F - (20.0\ cm)(7.35\ N) - (30.0\ cm)(0.980\ N) - (40.0\ cm)(4.90\ N) = 0$$

$$F = \frac{(20.0\ cm)(7.35\ N) + (30.0\ cm)(0.980\ N) + (40.0\ cm)(4.90\ N)}{8.00\ cm} = 46.55\ N$$

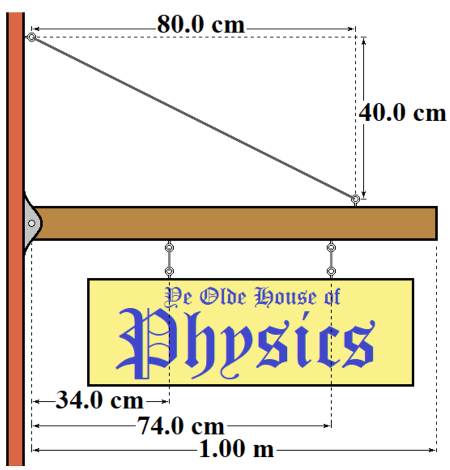
Once we have F we could use the sum of forces to get T, but for practice let's use another torque equation to get T.

To get T we need to move the axis of rotation. Let's place it at F.



$$\sum \tau = (8.00\ cm)T - (12.0\ cm)(7.35\ N) - (22.0\ cm)(0.980\ N) - (32.0\ cm)(4.90\ N) = 0$$

$$T = \frac{(12.0\ cm)(7.35\ N) + (22.0\ cm)(0.980\ N) + (32.0\ cm)(4.90\ N)}{8.00\ cm} = 33.3\ N$$



Example: A 2.50 kg sign is hung from an 11.0 kg beam as shown in the diagram. The sign is balanced so that half its weight is supported by each of the two wires above it. The beam is attached to the wall on the left by a frictionless pin that allows it to rotate and held in place by a guy wire above it. Determine the tension in the guy wire.

$$W_{beam} = m_{beam} g = (11.0\ kg)\left(9.80 \frac{m}{s^2}\right) = 107.8\ N$$

$$W_{sign} = m_{sign} g = (2.50\ kg)\left(9.80 \frac{m}{s^2}\right) = 24.5\ N$$

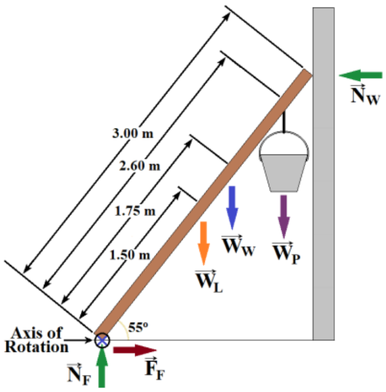
$$\sum \tau = T_y(80.0\ cm) - \frac{1}{2}(24.5\ N)(34.0\ cm) - \frac{1}{2}(24.5\ N)(74.0\ cm) - (107.8\ N)(50.0\ cm)$$

$$\sum \tau = T_y(80.0\ cm) - 416.5\ N \cdot cm - 906.5\ N \cdot cm - 5390.0\ N \cdot cm$$

$$\sum \tau = T_y(80.0\ cm) - 6713\ N \cdot cm = 0 \qquad T_y = \frac{6713\ N{\cdot}cm}{80.0\ cm} = 83.9125\ N$$

$$\theta = \tan^{-1}\left(\frac{40.0\ cm}{80.0\ cm}\right) = 26.565° \qquad T \sin\theta = T_y \qquad T = \frac{T_y}{\sin\theta} = \frac{83.9125\ N}{\sin 26.565°} = 188\ N$$

Example: A workman (weighing 755 N) leans a ladder (L = 3.00 m long, weighing 155 N) against a smooth (virtually frictionless) wall, making a 55.0° angle with the floor. He hangs a pail holding his work tools with a combined weight of 90.0 N from one of the top rails a distance d_1 = 2.60 m from the bottom of the ladder. The workmen then climbs up to do his work, balancing himself on a rail that is a distance d_2 = 1.75 m up the ladder. Determine the minimum value of the coefficient of static friction needed to keep the ladder from slipping.



When μ_S is at a minimum, then $F_F = F_{F-Max} = \mu_S N$.

$\sum F_x = F_F - N_W = 0 \qquad F_F = N_W$

$\sum F_y = N_F - W_L - W_W - W_P = 0$

$N_F = W_L + W_W + W_P = 155\,N + 755\,N + 90.0\,N = 1000\,N$

To get F_F or N_W we need to use a torque equation.

Place the axis of rotation where it will eliminate the most torques.

$$\sum \tau = N_W L \sin 55° - W_L\left(\frac{1}{2}L\right)\cos 55° - W_W d_2 \cos 55° - W_P d_1 \cos 55° = 0$$

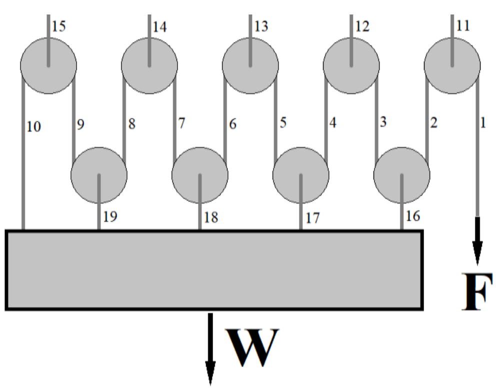
$$N_W L \sin 55° = W_L\left(\frac{1}{2}L\right)\cos 55° + W_W d_2 \cos 55° + W_P d_1 \cos 55°$$

$$N_W = \frac{W_L \cos 55°}{2 \sin 55°} + \frac{W_W d_2 \cos 55°}{L \sin 55°} + \frac{W_P d_1 \cos 55°}{L \sin 55°} \qquad N_W = \frac{W_L}{2 \tan 55°} + \frac{W_W d_2}{L \tan 55°} + \frac{W_P d_1}{L \tan 55°}$$

$$N_W = \frac{155\,N}{2 \tan 55°} + \frac{(755\,N)(1.75\,m)}{(3.00\,m)\tan 55°} + \frac{(90.0\,N)(2.60\,N)}{(3.00\,m)\tan 55°} = 417.2765\,N$$

$$\mu_S = \frac{F_F}{N_F} = \frac{N_W}{N_F} = \frac{417.2765\,N}{1000\,N} = 0.417$$

Example: A system of 9 frictionless massive pulleys is used to lift a heavy object (as shown). The 5 upper pulleys are anchored to the ceiling. The four lower pulleys are anchored to a 900 lb. box. How much force (F) is required to lift the box at a constant velocity?



Let's call T the tension in line 1. $T = F$.

The sum of the torques on the upper right pulley is zero. The tension in line 2 has to equal T, the tension in line 1.

The same logic works for lines 3 through 10. All those tensions are also T.

The sum of the forces on the upper right pulley is zero. The tension in line 11 has to equal 2T, the sum of the tension in lines 1 and 2.

The same logic works for lines 12 through 15, as well as 16 through 19. All those tensions are also 2T.

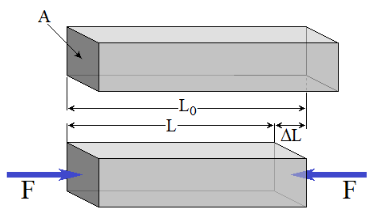
Sum of the forces on the box is zero: $T + 2T + 2T + 2T + 2T - W = 0$

$$9T = W \qquad T = \frac{W}{9} = \frac{900\ lbs}{9} = 100\ lbs$$

Deformation

- When an object changes shape due to applied forces, it is called Deformation.
- When the deformation exceeds The Elastic Limit, the change in shape becomes permanent. Until it reaches this limit, the changes are typically linear and the object will return to its original shape once the forces are removed.
- Elastic Deformation. $F = Y\left(\frac{\Delta L}{L_0}\right)A \qquad Y = \frac{Tensile\ Stress}{Tensile\ Strain} = \frac{F \cdot L_0}{A \cdot \Delta L}$

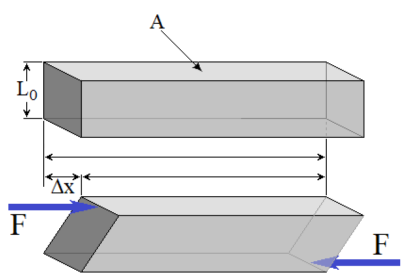
'Tensile' (from tension) refers to stretching. 'Compressive' refers to compression. Apart from that, these work much in the same way.



$Tensile\ Stress = \frac{F}{A} \qquad Tenstile\ Strain = \frac{\Delta L}{L_o}$

Young's Modulus (Y) is a property of the material (i.e. a constant that varies from material to material)

- Shear Deformation. $F = S\left(\frac{\Delta x}{L_0}\right)A \qquad S = \frac{Shear\ Stress}{Shear\ Strain} = \frac{F \cdot L_0}{A \cdot \Delta x}$

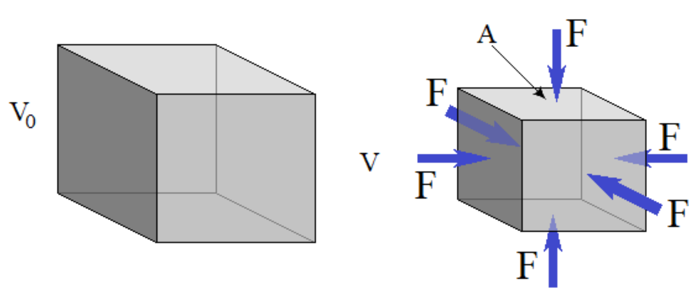


$$Shear\ Stress = \frac{F}{A}$$

$$Shear\ Strain = \frac{\Delta x}{L_o}$$

The Shear Modulus (S) is a property of the material (i.e. a constant that varies from material to material)

- Elastic Deformation. $\Delta P = -B\left(\frac{\Delta V}{V_0}\right) \qquad B = -\frac{Pressure\ Change}{Bulk\ Strain} = -\Delta P\frac{V_0}{\Delta V}$

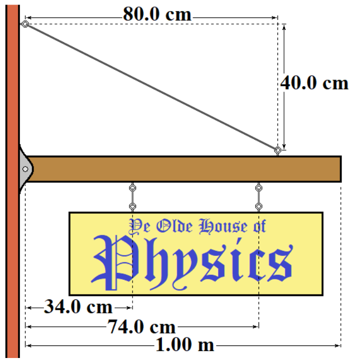


$$Pressure = \frac{F}{A}$$

$$Bulk\ Strain = \frac{\Delta V}{V_o}$$

The Bulk Modulus (B) is a property of fluids/gases (i.e. a constant that varies from material to material)

Example: An 89.4 cm long steel cable is used to hold up a beam and sign under a tension of 188 N. The cable radius is 5.00 mm, and the Young's modulus for steel is 2.00 x 10^{11} N/m^2. How much does the length of the cable change when the tension is added?



$$\Delta L = \frac{F \cdot L_0}{Y \cdot A} = \frac{F L_0}{Y \pi r^2}$$

$$\Delta L = \frac{(188\,N)(0.894\,m)}{\left(2.00 \times 10^{11}\,\frac{N}{m^2}\right)\pi(0.00500 m)^2}$$

$$\Delta L = 10.7\ \mu m$$